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$$u_1=\frac{e}{\sqrt{[(a^2e^2-d^2)(1-e^2)]}}>\frac{1}{a-d}.$$

Since the radius vector can have maxima and minima only at $\theta=0$, $\theta=\pm\theta_1$, and $\theta=\pi$, and since r_1 $[=1/u_1]<(a-d)<a+d$, the radius vector is a maximum at $\theta=0$ and $\theta=\pi$, and is a minimum at $\theta=\pm\theta_1$.

Since the ellipse is asymmetrical with respect to the lines $\theta=\pm\theta_1$, it follows from the statement made at the outset that the force is a multiple-valued function of the distance. This conclusion may be verified by deriving the law of force. Thus,

$$f=h^2u^2\left(u+\frac{d^2u}{d\theta^2}\right)=\frac{h^2\gamma^2(\beta\gamma+a^2-\beta)u^2}{[\beta u\pm a\sqrt{(\beta u^2+\gamma)}]^3}, \tag{6}$$

where h is the constant of areas, and the positive and negative signs are to be given as in (3). Thus, for values of r between r_1 and $(a-d)$, the required force is a multiple-valued function.

The force has one striking peculiarity. For $u>u_1$, $\beta u^2+\gamma<0$, so that the force is imaginary. From this fact it easily follows that the force is such as not to permit a real orbit anywhere within a circle of radius r_1 about the center of force.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

Note on Problem 266. While in Washington, D. C., during the latter part of last August, I called on Mr. Theodore L. DeLand of the United States Treasury Department. While there, he called my attention to the usage among practical computers and actuaries in England regarding the finite series. He showed me a number of books in which infinite series were used and in which it was assumed that the series is determined by the terms that are given.

Thus, in the series 1+7+12+21, it is assumed that the series has for its first differences, 5, 9, etc.; for the second differences, 4, 4, 4, etc. If only three terms were given, it would be assumed that the first differences are 5, 5, 5, etc. The series 1+3+7+17+... of which the sum of n terms were required becomes, with the understanding that its third differences are 2, 2, 2, etc., a definite series. I doubt, however, whether this convention adopted among many actuaries in England, is generally agreed to among mathematicians. Mr. DeLand based his solution of problem 266 on this assumption. Were this convention adopted, no ambiguity would arise in extending the series and finding its sum. ED. F.

GEOMETRY.

319. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

*Given the radii and the distances apart of the centers of three circles, to find the radii of the eight circles touching the three given circles.

Solution by the PROPOSER.

Let $AO=a$, $BP=b$, $CQ=c$, $AB=l$, $BC=m$, $CA=n$, $AD=q$, $BD=r$, $CD=p$, $\angle ADB=\theta$, $\angle BDC=\phi$, $\angle ADC=\psi$. Then $\cos\theta=\cos(\phi\pm\psi)$, and

$$\cos^2\theta+\cos^2\phi+\cos^2\psi-2\cos\theta\cos\phi\cos\psi=1.$$

$$\cos\theta=\frac{q^2+r^2-l^2}{2qr},\quad \cos\phi=\frac{r^2+p^2-m^2}{2rp},\quad \cos\psi=\frac{p^2+q^2-n^2}{2pq}.$$

Hence,
$$l^2(p^2-q^2)(p^2-r^2)+m^2(q^2-r^2)(q^2-p^2)+n^2(r^2-p^2)(r^2-q^2)$$
$$+l^2p^2(l^2-m^2-n^2)+m^2q^2(m^2-n^2-l^2)+n^2r^2(n^2-l^2-m^2)$$
$$+l^2m^2n^2=0\ldots(1).$$

I. In (1), let $p=\pm(c-x)$, $q=\pm(a-x)$, $r=\pm(b-x)$.

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*This problem, celebrated in the History of Mathematics, and also known as the "Tangency Problem," was first proposed and solved by Appolonius of Pergae, 200 B. C. Although this solution was lost for 1800 years, it was finally restored in 1600 A. D. by Vieta who, by reducing the original problem to a simpler form and thus solving simpler problems, gave an indirect solution.

The first direct solutions were furnished by Gaultier, 1813, and by Gergonne, 1814. The latter's method of solution is recorded in Carr's *Synopsis of Pure Mathematics*, page 224.

The first solution, though indirect, of its analogue in Solid Geometry, "To find a sphere touching four given spheres," was given by Fermat, 1665.

During the past century, numerous and varied geometrical solutions of this problem have appeared in many of the mathematical journals of Europe and America. The special problem, "Given the radii of three tangent circles to compute the radii of the two circles tangent to the three given circles," has been solved in various ways. From the very ingenious solution by Professor Enoch Beery Seitz, *School Visitor*, Vol. II, page 117, Mr. D. H. Davidson invented a method, *ibid*, Vol. VI, pages 80-84, of easily filling out a series of circles, beginning with any three given tangent circles.

The object of this solution is to complete the investigation by computing the radii of the eight circles touching any three circles, having given their radii and the distances apart of their centers. ZERR.

Two solutions of this problem are given in Vol. I, pages 220-222 of Leyborne's *Mathematical Questions*, and in Vol. IV, pages 259-275, are given Simson's, Vieta's, and Cauchy's solutions, together with a solution by Binet, and a trigonometric solution which appears to be by Leyborne himself, and a solution by Poncelet. A number of references and historical notes are there given. ED. F.